

Refutation of 0-1 laws in real-valued, probabilistic logics

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·, ∘ , ⊗ ; \ Not And ;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ←, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕ ;
% possibility, for one or some, ∃, ∃!, ◊, M ; # necessity, for every or all, ∀, □, L ;
(z=z) T as tautology, ⊤, ordinal 3 ; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero ;
(%z>#z) N as non-contingency, Δ, ordinal 1 ; (%z<#z) C as contingency, ∇, ordinal 2 ;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Notes: for clarity, we usually distribute quantifiers onto each designated variable; and for ordinal arithmetic, the result is implied.

From: Compton, K.J. (1988, 1999). 0-1 laws in logic and combinatorics.
researchgate.net/publication/239662281_0-1_laws_in_logic_and_combinatorics/link/00b4953232bc770fec000000/download kjc@umich.edu

Abstract This is a survey of logical results concerning random structures. A class of relational structures on which a (finitely additive) probability measure has been defined has a 0-1 law for a particular logic if every sentence of that logic has probability either 0 or 1. The measure may be an asymptotic probability on finite structures or generated on a class of infinite structures by assigning fixed probabilities to independently occurring properties. Conditions under which all sentences of a logic have a probability, and under which 0-1 laws occur, are examined. Also, the complexity of computing probabilities of sentences is considered.

2 Logics

transitive relation extending the relation θ. For example, consider the logic whose only extralogical symbol is a binary relation symbol E which we take to denote the edge relation on the class of graphs. The TC sentence

$$[X(x,y) \equiv \mathrm{TC}(E(x,y))]\, \forall x, y(x \neq y \rightarrow X(x,y))$$

asserts that a graph is connected. X interprets the path relation in each graph: it holds between two points precisely when there is a path between them.

(2.1.1)

LET p, q, r, s: x, y, z, E.
X is taken as x' for (p&(%s>#s)).
We ignore the symbol TC for transitive closure (logic), TCL.

(((p&(%s>#s))&(p&q))=(s&(p&q))) & ((p@q)>((p&(%s>#s))&(p&q))) ;

T**FF**C T**FF**N T**FF**C T**FF**N TCL (2.1.2)

Remark 2.1.2: Eq. 2.1.2 as rendered is *not* tautologous, to refute the sentence, denying transitive closure logic (TCL).

It is easy to see that LFP is at least as powerful as TC. For example, the transitive closure sentence above asserting connectivity in graph can be rewritten

$$\begin{array}{r}[X(x,y) \equiv E(x,y) \vee \exists z(X(x,z) \wedge X(z,y))] \\ \forall x, y(x \neq y \rightarrow X(x,y))\end{array}$$

in LFP.

(2.2.1)

(((p&(%s>#s))&(p&q))=((s&(p&q))+(((p&(%s>#s))&(s&%r))&
((p&(%s>#s)) & (%r&q))))) & ((#p@#q)>((p&(%s>#s))&(#p&#q))) ;
TCCC TCCC TCCN TCCN TCP (2.2.2)

Remark 2.2.2: Eq. 2.2.2 is *not* tautologous, to refute the sentence, denying least fixed point (LFP) logic. This further denies 0-1 laws in real-valued, probabilistic logics.

The following eight items are also denied: order theory, as particularly used by W. Oberschelp and R. Fagin; iterative logic (IT), refuted elsewhere here under induction and intensional logic; infinitary logic (IL), refuted elsewhere here under continuum hypothesis; class C structures; first-order FO 0-1 law logics; asymmetric probabilities of the foregoing; and the two real-valued logic classes below.

Most recently, these works are denied as based on the 1988 paper above:

Badia, G.; et al. (2022). New foundations of reasoning via real-valued first-order logics. arxiv.org/pdf/2207.00086.pdf

which is the follow on to:

Fagin, R. et al. (2021). Foundations of reasoning with uncertainty via real-valued logics. arxiv.org/pdf/2008.02429.pdf